12/6/16 "Hope is an accelerant."-Mrs. Lenoci

HW: "Implicit Differentiation 4" w/s #10, 25, 26
Test 2 on Tuesday 12/20

AIM: How do we find where there are horizontal tangents using Implicit Differentiation?

Warm Up:

Product Product

1) Determine $\frac{dy}{dx}$ of $x^2y + 3xy^3 - x = 3$

$$x^2\frac{dy}{dx} + 2xy + 3x3y^2\frac{dy}{dx} + 3y^3 - 1 = 0$$

$$-2xy \qquad -3y^3 + 1 \qquad -2xy-3y^3+1$$

$$x^2\frac{dy}{dx} + \underbrace{3x3y^2}_{9xy^2}\frac{dy}{dx} = -2xy-3y^3+1$$

$$\frac{\frac{dy}{dx}\left(x^2+9xy^2\right)}{x^2+9xy^2} = \frac{-2xy-3y^3+1}{x^2+9xy^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy-3y^3+1}{x^2+9xy^2}}$$

2) Find the equation(s) of the tangent line to the curve from #1 when y=1

$x^2y + 3xy^3 - x = 3$

Point:

$x^2(1) + 3x(1)^3 - x = 3$

$x^2 + 3x - x = 3$

$\quad\quad\quad -3 \quad -3$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

Points:

$(-3,1) \quad (1,1)$

Slope:

$$\frac{dy}{dx} = \frac{-2xy - 3y^3 + 1}{x^2 + 9xy^2}$$

$(-3,1)$

$$\frac{dy}{dx} = \frac{-2(-3)(1) - 3(1)^3 + 1}{(-3)^2 + 9(-3)(1)^2}$$

$$\frac{dy}{dx} = \frac{4}{-18} = -\frac{2}{9}$$

$$\boxed{y - 1 = \frac{-2}{9}(x+3)}$$

$(1,1)$

$$\frac{dy}{dx} = \frac{-2(1)(1) - 3(1)^3 + 1}{(1)^2 + 9(1)(1)^2}$$

$$\frac{dy}{dx} = \frac{-4}{10} = -\frac{2}{5}$$

$$\boxed{y - 1 = \frac{-2}{5}(x-1)}$$

HW check:

product

$$x^2 + y^2 - xy + 3x - 9 = 0$$

21) $2x + \boxed{2y\frac{dy}{dx}} \boxed{- x\frac{dy}{dx}} - y + 3 = 0$

$-2x \quad\quad +y \quad -3 \quad\quad -2x+y-3$

$$2y\frac{dy}{dx} - x\frac{dy}{dx} = -2x + y - 3$$

$$\frac{\frac{dy}{dx}(2y-x)}{2y-x} = \frac{-2x+y-3}{2y-x}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x+y-3}{2y-x}}$$

22) Horizontal tangents?

$\frac{dy}{dx} = 0$ Numerator $= 0$

$$-2x + y - 3 = 0$$

$+2x \quad +3 \quad\quad +2x+3$

relationship when the tangent is horizontal $\rightarrow$ $y = 2x+3$

sub in for y in the original equation

$$x^2 + (2x+3)^2 - x(2x+3) + 3x - 9 = 0$$

$$x^2 + 4x^2 + \boxed{12x} + \cancel{9} - 2x^2 \cancel{-3x} \cancel{+3x} \cancel{-9} = 0$$

$$3x^2 + 12x = 0$$

$$3x(x+4) = 0$$

$x = 0$ | $x = -4$

Plug these into the relationship equation or the original to get the y values.

$y = 2(0)+3$ $\quad$ $y = 2(-4)+3$

$y = 3$ $\quad$ $y = -5$

$\boxed{(0,3)}$ $\quad$ $\boxed{(-4,-5)}$